



11th International Academic Conference

"Global Goals, Local Actions: Looking Back and Moving Forward 2020"

Critical Thinking in Mathematics: A Must-Thinking Process to be Developed in Mathematics Classroom

Pongsakorn Plubsiri¹, Supotch Chaiyasang²

¹Master Student, Mathematics Education Program

Suan Sunandha Rajabhat University

²Assistant Professor, International College

Suan Sunandha Rajabhat University

Abstract

This paper aims to present critical thinking in mathematics which is a part of the authors' research. Critical thinking in mathematics is related to thinking, reasoning, and critical thinking. The authors define critical thinking in mathematics in four aspects which are the following: 1) building mathematical ideas through explaining, questioning, and organizing, 2) using mathematical understanding to generate, and create new ideas, 3) providing reasons or judgments, and 4) performing ways to solve mathematical problems. The authors also present the related components of it which are questioning, reasoning, and problem solving.

Keywords: Critical Thinking in Mathematics, Questioning, Problem Solving

This paper aims to present the concept of critical thinking in mathematics and its related study. The sequence of this presentation is introduction, thinking and reasoning, critical thinking, critical thinking in mathematics, and some related components.

Introduction

Critical thinking skill is very crucial in 21st century which is technology and information era. Many papers stated about 21st century skills which are necessary to be developed in our students. The figure below presents 21st century knowledge and skills rainbow which gives us a wider view of concerns in educating our students.

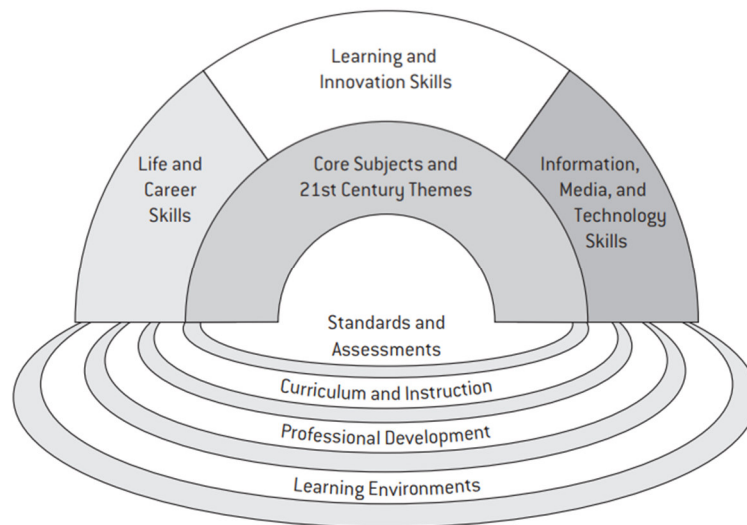


Figure 1: The 21st Century Knowledge-and-Skills Rainbow.

Source: Reasoning and Problem Solving: A Handbook for Elementary School Teachers (Trilling & Fadel, 2009, p. 173)

From the figure, in core subjects and 21st century themes, a group of skills needed to learn is learning and innovation skills. That is students must possess both learning skills and innovation skills. Self-learning skill is becoming a must for students in this century. Learning in classroom is changing. Students need to find out resources for their lessons, for classroom assignment, and for their curiosity. For those learning activities, students' abilities to select resources, to discuss with their friends, to reason, to organize resources and to solve problems are necessary. All of those activities are concerned with thinking and reasoning.

Purpose of the Study

To find the components of critical thinking in mathematics.

Scope of the Study

This research focused on critical thinking in mathematics. The data and literature are from books about critical thinking, journals, articles, NCTM (National Council of Teachers of Mathematics) yearbooks, and Principles and Standards for School Mathematics (NCTM, 2000).

Results

Thinking and Reasoning

Thinking is the ability of a person to reach a valid conclusion from a given set of data. Reasoning is a part of thinking that lies beyond recall level. The figure below shows a hierarchy of thinking, reasoning, and higher-order thinking.

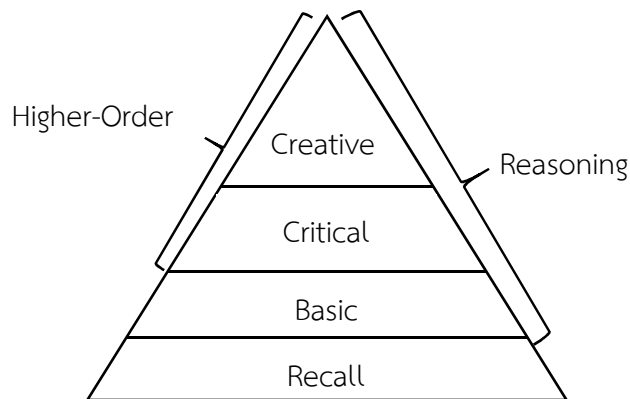


Figure 2: Hierarchy of Thinking

Source: Reasoning and Problem Solving: A Handbook for Elementary School Teachers (Krulik & Runick, 1993, p. 3)

Reasoning is divided into three levels: Basic, Critical, and Creative. According to Krulik and Rudnick, Basic Thinking requires thinking that understands concepts and apply these concepts in everyday life; Critical Thinking requires thinking that examines, relates, and evaluates aspects of the problems; Creative Thinking requires thinking that is original and effective and produces complex concepts (Krulik & Rudnick, 1993, pp. 4-5).

Critical Thinking

Critical thinking is a process that relies upon and develops a wide range of skills and personal qualities. Like other forms of thinking, it is improved by practicing. Critical thinking has been identified its components by many researchers.



Cottrell (2005) stated that critical thinking is a complex process of deliberation which involves a wide range of skills and attitudes. Students are expected to develop critical thinking skills so that they can dig deeper below the surface of the subjects they are studying and engage in critical dialogue with its main theories and arguments. This is usually through engaging in critical debate, in seminars, in presentations, or in writing produced for assessment or publication.

Students need to develop the ability to critically evaluate the work of others. While some find this easy, others tend to accept or apply the results of other people's research too readily, without analyzing it sufficiently to check that the evidence and the reasoning really support the main points being made. Cottrell (2005) described critical thinking as a student means in:

1. finding the best evidence
2. evaluating the strength of that evidence
3. finding ways to the conclusion from the evidence
4. constructing line of reasoning to others
5. selecting the best example
6. providing evidence to illustrate an argument.

Bodner (1988), for example, describes chemistry students as being unable to apply their knowledge outside the narrow domain in which it was learned. They know without understanding. Bodner suggests that, instead of focusing primarily on standard chemical calculations in books, students should be looking for answers to questions such as how do we know? and why do we believe?

Fisher (1995) suggested critical thinking is to explain what someone is thinking. Learning to think critically is concerned about:

1. learning how to ask, and when to ask
2. what's the question
3. how to reason
4. when to use reasoning
5. what reasoning methods which use.

Ennis (1996) suggested that critical thinking is a process that aims so that we can make decisions that make sense, so what we think is the best of a truth we can do it right.



Critical Thinking in Mathematics

Many experts define the term critical thinking in mathematics with different points of view but they identified some common aspects such as questioning about problems, analyzing facts, organizing ideas, explaining and defending opinions, and solving problems.

Chance (1986), in his book, *Critical Thinking: An Overview*. Educational Psychology Interactive, defined critical thinking (which is suitable to apply in mathematics) as the ability to analyze facts, generate and organize ideas, defend opinions, make comparisons, draw conclusions, evaluate arguments and solve problems.

Krulik and Rudnick (1999) in NCTM (National Council of Teachers of Mathematics) 1999 yearbook article, *Innovative Task to Improve Critical - and Creative - Thinking Skills*, stated critical thinking in mathematics as the following:

Critical thinking is thinking that examines, relates, and evaluates all aspects of the situation or problem. It includes gathering, organizing, remembering, and analyzing information. Critical thinking includes the ability to read with understanding and to identify extraneous and necessary material. It also means being able to draw proper conclusion from a given set of data and being able to determine inconsistencies and contradictions in a set of data. Critical thinking is analytical and reflexive.

Based on the opinions of experts, and their studies, the authors of this paper summarized the key components of critical thinking in mathematics for their research as following.

Critical thinking in mathematics is students' abilities in:

1. building mathematical ideas through explaining, questioning, and organizing
2. using mathematical understandings to generate, and create new ideas
3. providing reasons or judgments
4. performing ways to solve mathematical problems.

From the above summarizing, the authors would like to describe some key components and their related studies which are questioning, reasoning, and problem solving.

Questioning

Questioning is a technique of teaching that commonly used at all grade levels. Teachers used questioning to check, to review on learning, to pose a problem, and to challenge students to reflect their knowledge (Wilén, 1987). Questioning is an important component of the learning process. Good questioning techniques are a fundamental instructional tool for effective teachers (Wilén & Clegg, 1986). Herr (2007) stated questioning is



important to clarify thinking and understanding, to develop thinking skill, to establish links between ideas, and to obtain feedback on learning. There are many types of questions in mathematics and science classes such as Socratic method, and open-ended questions. Questioning on Socratic method can be classified as the following (Paul, Wilsen & Binker, 1993):

1. questioning of clarification
2. questioning that probe assumption
3. questioning that probe reason and evidence
4. questioning about viewpoint or perspectives
5. questioning that probe implication and consequences
6. questioning about the question.

For open-ended questions, they have been developed in 1970's in Japan (Nohda, 1991). Open-ended questions are the questions formulated to answers in many ways and asked to focus in different methods, ways, or approaches to getting answers (Isoda, 2007). The open-ended questions were categorized into three types as the following:

1. finding relation questions: students are asked to find some mathematical rules or relations
2. classifying questions: students are asked to classify according to characteristics
3. measuring questions: students are asked to measure a certain phenomenon (Sawada, 1997).

Reasoning

Reasoning is an essential tool for mathematics and everyday life. Reasoning could separate into three types which are: 1) intuitive reasoning, 2) inductive reasoning and 3) deductive reasoning. Intuitive reasoning is an ability to understand something with some information. Inductive reasoning is a method of reasoning that finding to generalize problems. Deductive reasoning is drawing a conclusion from given data (Baroody, 1993).

Principles and Standards for School Mathematics mentioned about standard for reasoning and proof from kindergartens to grade 12 as follows:

1. recognize reasoning and proof as fundamental aspects of mathematics
2. make and investigate mathematical conjectures
3. develop and evaluate mathematical arguments and proofs
4. select and use various types of reasoning and method of proof (NCTM, 2010).

Krulik and Rudnick (1993) in his book, Reasoning and Problem Solving: A Handbook for Elementary School Teachers, presented a workable set of heuristic that can form the basis of

an instructional program in reasoning. The flowchart below presented the set of heuristics that every person should use when he confronted by a situation that requires solution. The more experienced the reasoner, the more automatic the back-and-forth motion becomes.

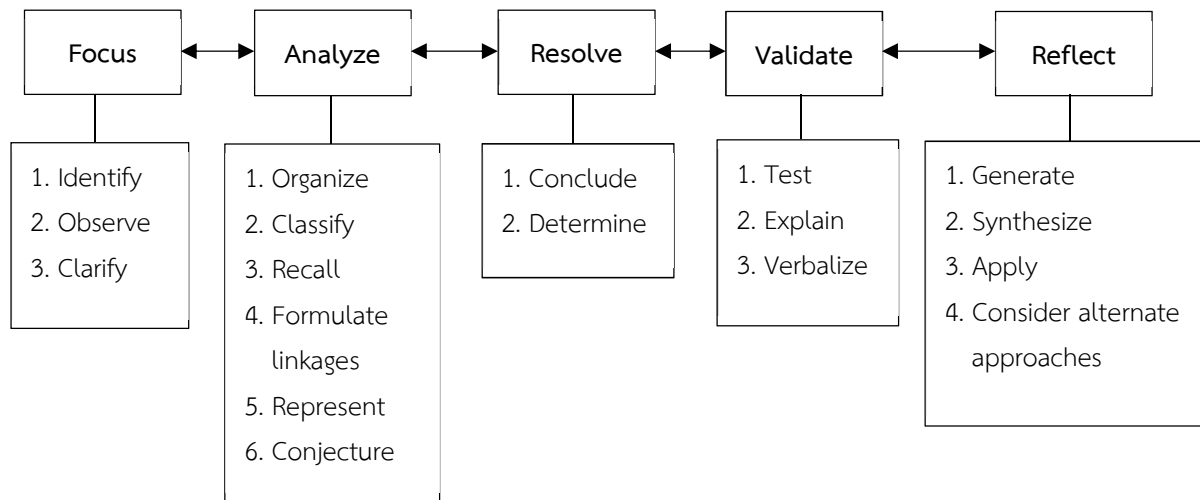


Figure 3: The Heuristic of Reasoning

Source: Reasoning and Problem Solving: A Handbook for Elementary School Teachers (Kruik & Runick, 1993, p. 28)

Problem Solving

In the year 1980, NCTM issued its' yearbook, An Agenda for Action: Recommendations for School Mathematics of the 1980s. The yearbook stated the importance of problem solving that it is the heart of mathematics learning. In the same year, NCTM collected research in mathematics education, Research in Mathematics Education. It presented a long chapter on problem solving where the chapter presented the following: 1) meaning of problem solving, 2) students' behavior during solving problems, and 3) teaching problem solving.

NCTM (2000), in problem solving standard from Principles and Standards for School Mathematics, stated that students from kindergartens to grade 12 should be able to:

1. build new mathematical knowledge through problem solving
2. solve problems that arise in mathematics and in other contexts
3. apply and adapt a variety of appropriate strategies to solve problems
4. monitor and reflect on the process of mathematical problem solving.

Posamentier and Kruik (1998), in his book, Problem-Solving Strategies for Efficient and Elegant Solutions: A Resource for Mathematics Teacher, suggested various strategies and examples in solving problems as follow:



1. working backwards
2. finding a pattern
3. adopting a different point of view
4. solving a simpler, analogous problem
5. considering extreme cases
6. making a drawing
7. intelligent guessing and testing
8. accounting of all possibilities
9. organizing data
10. logical reasoning.

In teaching problem solving, Pólya (1957), in his book *How to Solve It*, distinguished four steps of solving a problem:

1. understand the problem
2. devise a plan
3. carry out the plan
4. look back.

Pólya mentioned that in solving a problem, students need not orderly follow the steps.

Schoenfeld (1985), in his outstanding research, *Mathematical Problem Solving*, discovered that basic mathematics knowledge, flexibility, and efficiency are necessary for problem solving. He categorized behaviors into four groups as follow:

1. resource: mathematical knowledge possessed by the individual that can be brought to bear on the problem at hand
2. heuristic: strategies and techniques for making progress on unfamiliar or nonstandard problems; rule of thumb for effective problem solving including drawing figure, exploiting related problems, reformulating problems
3. control: global decision regarding the selection and implementation of resources and strategies
4. belief system: one's mathematical world view, the set of (not necessarily conscious) determinants of an individual's behavior.

In grading student performance in solving mathematics problems Charles, Lester, and O'Daffer (1987) in his book *How to Evaluate Progress in Problem-Solving* suggested two methods in scoring, analytic scoring scales and holistic scoring scales. The table below showed the scoring by using analytic scoring scales.



Table 1: The analytic scoring scale

Understand the problem	2	Complete understanding of the problem
	1	Part of the problem misunderstood or mis interpreted
	0	Complete misunderstanding of the problem
Planning a solution	2	Plan could have led to a correct solution if implemented correctly
	1	Partially correct plan based on part of the problem being interpreted correctly
	0	No attempt, or totally inappropriate plan
Getting an answer	2	Correct answer
	1	Copying error; computational error; partial answer for a problem with multiple answers
	0	No answer, or wrong answer based on an inappropriate plan

Source: How to Evaluate Progress in Problem-Solving, (Charles, Lester, & O'Daffer, 1987)

Conclusion and Discussion

From the study, researchers found that there are many components that were important to improve students' critical thinking in mathematics. Those were questioning, reasoning, and problem solving.

Questioning is a technique that a teacher asks students in a wide range of questions. It was used to lead students a deeper understanding and thinking critically with reasons (Nappi, 2007).

Reasoning is a part of critical thinking (Fisher, 1995; Cottrell, 2005). In mathematics, reasoning is also used to solve the problem (Krulik & Runick, 1993).

Problem solving is skills to use mathematics knowledge and process to solve problems (NCTM, 2000).

For practical use in this study, the researchers classified the components of critical thinking in mathematics as the following.

Critical thinking in mathematics is students' abilities in:

1. building mathematical ideas through explaining, questioning, and organizing
2. using mathematical understandings to generate, and create new ideas
3. providing reasons or judgments
4. performing ways to solve mathematical problems.



11th International Academic Conference

"Global Goals, Local Actions: Looking Back and Moving Forward 2020"

Reference

- Baroody, A. J. (1993). *Problem Solving, Reasoning, and Communication, K-8: Helping Children think mathematically*. Englewood Cliffs: NJ, Macmillan Publishing Company.
- Bodner, G. M. (1988). Consumer Chemistry: Critical Thinking at the Concrete Level. *Journal of Chemistry Education*, 65(3), 212-13.
- Chance, P. (1986). *Thinking in The Classroom: A Survey of Programs*. New York: Teachers College, Columbia University.
- Charles, R., Lester, F., & O'Daffer, P. (1987). *How to evaluate progress in problem-solving*. Reston, VA: NCTM.
- Cottrell, S. (2005). *Critical Thinking Skill: Developing Effective Analysis and Argument*. NY: Palgrave Macmillan.
- Ennis, R. H. (1996). *Critical Thinking*. USA: Prentice Hall, Inc.
- Fisher R. (1995). *Teaching Children to Learn*, Stanley Thornes.
- Herr, N. (2007). *The Sourcebook for Teaching Science*. San Francisco, CA: Jossey-Bass Publishers.
- Isoda, M. (2007). Section 2: A Brief History of Mathematics Lesson Study in Japan. In M. Isoda, M. Stephens, Y. Ohara, & T. Miyakawa (Eds.), *Japanese Lesson Study in Mathematics Its Impact, Diversity and Potential for Educational Improvement* (pp.8-15). Singapore: World Scientific Publishing.
- Krulik, S., & Rudnick, J. A. (1993). *Reasoning and Problem Solving: A Handbook for Elementary School Teachers*. Needham Heights, MA: Allyn and Bacon.
- _____. (1999). Innovative Task to Improve Critical – and Creative Thinking Skills. In L. V. Stiff & F. R. Curcio (Eds.) *Developing Mathematical Reasoning in Grade K–12, 1999 Yearbook* (pp. 138-145). Reston, VA: NCTM.
- Nappi, J. S. (2017). The Importance of Questioning in Developing Critical Thinking Skills, The Delta Kappa Gamma Bulletin. *International Journal for Professional Educators*, 1(30), 30-41.
- NCTM. (1980). *An Agenda for Action: Recommendations for School Mathematics of the 1980s*. Reston, VA: NCTM.



11th International Academic Conference

"Global Goals, Local Actions: Looking Back and Moving Forward 2020"

- NCTM. (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.
- Nohda, N. (1991). Paradigm of The “Open-Approach” Method in Mathematics Teaching: Focus on Mathematical Problem Solving. *International Reviews on Mathematical Education*, 23(2), 32–37.
- Panadero, E., Jonsson, E. (2013). The Use of Scoring Rubrics for Formative Assessment Purposes Revisited: A Review. *Educational Research Review*, 9, 129-144.
- Paul, R., Willson, J., & Binker, A. J. A. (1993). *Critical Thinking: What Every Person Needs to Survive in A Rapidly Changing World* (3rd ed.). Santa Rosa, CA: Foundation for Critical Thinking.
- Pólya, G. (1957). *How to Solve It: A New Aspect of Mathematical Method*. Princeton, NJ: Princeton University Press.
- Posamentier, A. S., Krulik, S. (1998). *Problem-Solving Strategies for Efficient and Elegant Solutions: A Resource for Mathematics Teacher*. CA: Crowin Press.
- Sawada, T. (1997). Developing Lesson Plans. In S. Shimada, & J.P. Becker. *The open-ended approach: A New Proposal for Teaching Mathematics* (pp.23-35). Virginia: National Council of Teachers of Mathematics.
- Schoenfeld, A. H. (1985). *Mathematical Problem Solving*. NY: Academic Press.
- Trilling, B., & Fadel, C. (2009). *21st Century Skills: Learning for Life in Our Times*. San Francisco, CA: John Wiley & Sons.
- Wilén, W. W. (1987). *Questions, Questioning Techniques, and Effective Teaching*. DC: National Education Association.
- Wilén, W. W., & Clegg, A. A. (1986). Effective Questions and Questioning: A research review. *Theory and Research in Social Education*, 14(2), 153–161.